PROJECTED WRITTEN NOTES FROM THE M325K LECTURE ON THURSDAY, FEBRUARY 1, 2024, ON DEDUCTIONS, ON WRITING PROOFS, ON DEFINITIONS AND ON THE DIRECT PROOF METHOD.

CLASS #6

Deductions; Given a list of premises and given a "statement to be deduced";

a Deduction is a list of Standard Valid Arguments such that

1. For each argument in the list, each premise of that argument either comes from the given list of premise OR appears as a conclusion reached in an argument earlier in the list.

2. The Conclusion of the last argument in the list is the given "statement to be deduced", given at the start.

An EXAMPLES Deduction.

The Given Premise List

(a). $p \vee q$

(b) $q \to r$

(c) $\sim r$

Req'd

The given "statement to be deduced":

p

Req'd

STATEMENTS		JUSTIFICATIONS
(1) $q \to r$	Req'd	by Premise (b)
$\sim r$		by Premise (c)
$\therefore \sim q$	Req'd	by Modus Tollens
(2) $p \vee q$		by Premise (a)
$\sim q$		by Argument (1)
$\therefore p$		by Elimination

Done

We call "p" it "a valid conclusion from the premise list".

$$\left.\begin{array}{l} (a)\ p \vee q \\ (b)\ q \to r \\ (c)\ \sim r \\ \hline \therefore\ p \end{array}\right\}$$ This is a Valid Argument.

PROOF WRITING Sec's 4.1 & 4.2

Every proof must have a heading of the form:

To Prove: (The statement to be proved).

proof: ______
______
______
______

(Req'd last line) Therefore, (∴) (The statement to be proved) by ______. ■ (the proof method used)

END-OF-PROOF DELIMITER.

Ex: ■, QED, ☺

Assumptions

- In this text we assume a familiarity with the laws of basic algebra, which are listed in Appendix A.
- We also use the three properties of equality: For all objects A, B, and C, (1) $A = A$, (2) if $A = B$ then $B = A$, and (3) if $A = B$ and $B = C$, then $A = C$.
- In addition, we assume that there is no integer between 0 and 1 and that the set of all integers is closed under addition, subtraction, and multiplication. This means that sums, differences, and products of integers are integers.
- Of course, most quotients of integers are not integers. For example, $3 \div 2$, which equals $3/2$, is not an integer, and $3 \div 0$ is not even a number.

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NUMBER THEORY :

The Study of the Integers ($\mathbb{Z}$) and how they operate together.

We will be using techical terms with definitions.

When applying a definition of a term in a proof, you must use the exact wording used in the definition given in the book.

We look at the Definitions of "EVEN NUMBER" and "ODD NUMBER" Given in the book (as shown in the handout).

In-the-Book Definitions (I)

The "In-the-Book Definitions" are the definitions given in the book of terms, some of which are well known and understood already with perhaps other simpler and equivalent definitions. However, the "In-the-Book Definitions" are the definitions which are necessary for using these terms in a proof. Although the simpler definitions learned in the past are useful for understanding the objects defined, these simpler definitions are often very difficult to use when writing a proof of a theorem which involves these objects.

Since the "In-the-Book Definitions" are the definitions to be used in proofs, it is important to memorize these definitions word for word. One advantage to memorizing these definitions is that, when one of these definitions is applied within a proof, the exact wording of the "In-the-Book Definition" can be used directly in the wording of the proof.

Definition: Let n be an integer .

Integer n is an even integer $\Leftrightarrow$ there exists an integer k such that $n = 2k$.

Integer n is an odd integer $\Leftrightarrow$ there exists an integer k such that $n = 2k + 1$.

It can be proved that every integer is either even or odd and that no integer is both even and odd. The even-ness or odd-ness of an integer is called its parity .
The terms even number and odd number mean "even integer" and "odd integer," respectively.

Definition: Let n be an integer .

Integer n is prime $\Leftrightarrow$

$(n > 1)$ AND

(For all positive integers r , s , IF $n = rs$, THEN $(r = 1$ OR $s = 1))$.

Integer n is composite $\Leftrightarrow$

$(n > 1)$ AND (there exist integers r and s such that

$1 < r < n$ and $1 < s < n$ and $n = rs$) .

The number 1 is neither prime nor composite.

It can be proved that every integer > 1 (which is greater than 1) is either prime or composite and no integer is both prime and composite.

The terms prime number and composite number mean "prime integer" and "composite integer," respectively.

EXAMPLES OF THE TWO WAYS TO APPLY A DEFINITION IN A PROOF.

① [PROVE THAT 15 IS ODD]

Let $k = 7$.

$\therefore$ k is an integer since 7 is an integer.

$15 = 2 \times 7 + 1$ by Rules of Algebra.

$15 = 2k+1$, by substitution.

$15 = 2k+1$ and k is an integer.

$\therefore$ 15 is odd, by definition of "odd".

② Let s be an integer.

Suppose that s is odd.

[We conclude that s has the properties of the def'n of "odd"]

$\therefore$ By def'n of "odd", there exists an integer t such that
$s = 2t+1$.

etc

Methods of Proof.

Proving Existential Statements : $\exists x \in D$ such that $P(x)$.

Two Ways

① Give a particular example.
(see the proof that 15 is odd, shown above)

② Derive a formula that will produce an example $x \in D$ such that $P(x)$, (A formula like: "Let $x = \underline{4t+5}$".

(An Example of this is later in the notes.)

Proving Universal Statements : "$\forall x \in D, P(x)$"

Two Ways!

① The method of Exhaustion. (ONLY For when Domain is Finite and small)

For Say, $D = \{x_1, x_2, x_3\}$.

① Show $P(x_1)$ is true.
② Show $P(x_2)$ is True
③ show $P(x_3)$ is True.

$\therefore$ For all $x \in D$, $P(x)$, by Proof-by-Exhaustion QED.

(2) The Direct Proof Method to prove "$\forall x \in D, P(x)$."

To Prove: $\forall x \in D, P(x)$.

Proof: (1) Define a variable, say x, to represent an arbitrary, but particular, value in D

(2) SHOW That $P(x)$ is true about the arbitrary, but particular, value x.

(3) Conclude that $P(x)$ is true for all $x \in D$, by Direct Proof. QED

Note: You Disprove a universal statement by proving the existance of a counterexample.

A Direct Proof EXAMPLE

To Prove: For every integer n,

$4n+6$ is even.

Proof: Let n be any integer.

"NEED TO SHOW" → [N.T.S.: $4n+6$ is EVEN]

$4n+6 = 2(2n+3)$ by Rules of Algebra. (ROA).

Let $k = 2n+3$ and

k is an integer because sums and products of integers are integers.

$\therefore$ $4n+6 = 2k$, by substitution.

$\therefore$ $4n+6$ is Even, by definition of "even".

$\therefore$ For every integer n, $4n+6$ is even, by Direct Proof.

QED

(Quod erat demonstrandum)

FOR THE REMAINDER OF
THE PROJECTED WRITTEN NOTES,
Closely read the following two handouts:

1. "THE METHOD OF DIRECT PROOF"

and

2. "GROUND RULES (AND OTHER Rules)
of PROOF WRITING.

These handouts can be found ONLINE,
(THROUGH CANVAS)
IN THE HANDOUTS FOLDER CALLED
"BASIC PROOFS."